# The Burr XI Generated Family of Distribution with Illustration to Cancer Patients Data

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## ABSTRACT

The Burr system of distributions appeared in (Burr, 1942). On the other hand in (Eugene et al., 2002), they pioneered the beta-generated method of creating probability distributions. Inspired by these developments, we introduce the Burr XI generated method of creating probability distributions. We show a distribution arising from this method is a good fit to real-life data, indicating the new method of creating probability distributions should be applicable in various disciplines. Finally, we ask the reader to investigate some properties and applications of a so-called Burr VI generated family of distributions of two types.

***Keywords:*** *Burr System of Distributions, Breast Cancer, T-X(W) Family Of Distributions, Beta-G*

## 1. Introduction and the New Family

The Burr system of distributions (Burr, 1942) arise from the differential equation

where

$$y' = y(1-y)g(x,y)$$

$$y' = \frac{dy}{dx} = \frac{dF(x)}{dx} = f(x)$$

$$y = F(x)$$

and g(x, y) is a nonnegative function for 0 ≤ y ≤ 1 and x is in the range over which the solution is to be used. The Burr XI distribution has CDF, and for example, see Table 1.1 (Momanyi, 2017), given by

$$F(x) = \left( x - \frac{\sin(2\pi x)}{2\pi} \right)^r$$

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where $r > 0$, and $0 < x < 1$. By differentiating the CDF above, we see the PDF of the Burr XI distribution is given by

$$f(x) = 2r\sin^2(\pi x)\left(x - \frac{\sin(2\pi x)}{2\pi}\right)^{r-1}$$

Now we introduce the Burr XI generated family of distributions via the following integral

$$\int_0^{G(x;\xi)} 2r\sin^2(\pi t)\left(t - \frac{\sin(2\pi t)}{2\pi}\right)^{r-1} dt$$

where $r > 0$ and $G(x;\xi)$ is the CDF of some baseline distribution. The parameter space of $\xi$ depends on the choosen baseline distribution. Evaluating this integral leads to the following

Proposition 1.1. *The CDF of the Burr XI-G family of distributions is given by*

$$K(x;\xi,r) = \left(G(x;\xi) - \frac{\sin(2\pi G(x;\xi))}{2\pi}\right)^r$$

*where $x \in \mathbb{R}$, $r > 0$, and $G(x;\xi)$ is the CDF of some baseline distribution. The parameter space of $\xi$ depends on the choosen baseline distribution.*

By differentiating the CDF above, we have the following

Proposition 1.2. *The PDF of the Burr XI-G family of distributions is given by*

$$k(x;\xi,r) = 2rg(x;\xi)\sin^2(\pi G(x;\xi))\left(G(x;\xi) - \frac{\sin(2\pi G(x;\xi))}{2\pi}\right)^{r-1}$$

*where $x \in \mathbb{R}$, $r > 0$, $G(x;\xi)$ and $g(x;\xi)$ are the CDF and PDF, respectively, of some base- line distribution. The parameter space of $\xi$ depends on the choosen baseline distribution*

The rest of this paper is organized as follows. In the next section we show a so-called Burr XI-Weibull distribution is a good fit to the breast cancer data (Girish and Jayakumar, 2017). The last section is devoted to the conclusions. As a further recommendation we suggest obtaining some properties and applications of a so-called Burr VI generated family of distributions of two types.

## 2. Practical Illustration

We assume the baseline distribution is Weibull distributed with the following CDF

$$G(x;a,b) = 1 - e^{-\left(\frac{x}{b}\right)^a}$$

where x, a, b > 0. By Proposition 1.1, we have the following

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Corollary 2.1. *The CDF of the Burr XI-Weibull distribution is given by*

$$K(x;a,b,r)=\left(-e^{-\left(\frac{x}{b}\right)^{a}}+\frac{\sin(2\pi e^{-\left(\frac{x}{b}\right)^{a}}}{2\pi}+1\right)^{r}$$

*where* $x, a, b, r > 0$

Notation 2.2. *We write* $V \sim BXIW\ (a,\ b,\ r)$, *if V is a Burr XI-Weibull random variable.*

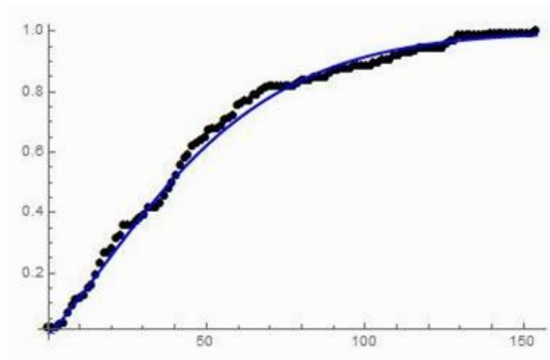


**Figure 1:** The CDF of *BXIW* (1.20333, 97.7151, 0.329319) fttted to the empirical distribution of the breast cancer data (Girish and Jayakumar, 2017)

Remark 2.3. *The PDF of the Burr XI-Weibull distribution can be obtained by differen- tiating the CDF*

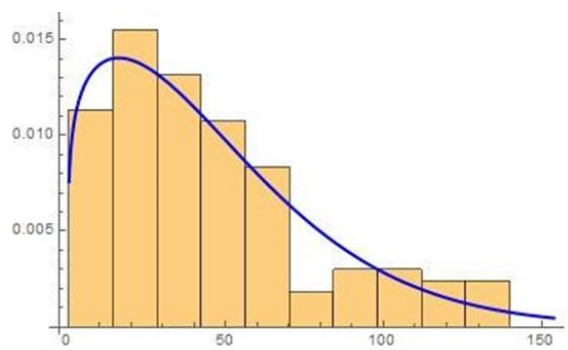


**Figure2:** ThePDFof*BXIW*(1.20333,97.7151,0.329319)fttted to the histogram of the breast cancer data (Girish and Jayakumar, 2017)

## 3. Concluding Remarks and Further Recommendations

The present paper has introduced a family of distributions arising from the Burr system of distributions. In particular, a so-called Burr XI-G family of distributions have been introduced, and a member of this class of distributions is shown to be a good fit to real life data in the health sciences. The Burr VI distribution has CDF

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$$Z(x;k,c,r)=(ke^{-c\sinh(x)}+1)^{-r}$$

where *k, c, r* > 0 and $x\in\mathbb{R}$. Inspired by the *T* - *X*(*W*) framework (Alzaatreh and Lee, 2013b), we ask the reader to investigate some properties and applications of a so-called Burr VI generated family of distributions of two types. We leave the reader with the following integral representations for their CDF's.

Definition 3.1. *The CDF of the Burr VI generated family of distributions is given by*

$$Q(x;\xi,k,c,r)=\int_{-\infty}^{W(G(x;\xi))}\frac{ckr\cosh(t)(ke^{-c\sinh(t)}+1)^{-r}}{e^{c\sinh(t)}+k}dt$$

*where G*(*x*; *ξ*) *is the CDF of some baseline distribution. The parameter space of ξ and x depends on the chosen baseline distribution, and k, c, r* > 0*.*

(a) *The Burr VI generated family of distributions is of type I, if we take*

$$W(G(x;\xi))=log(-log(1-G(x;\xi)^{\alpha}))$$

*for some α* > 0

(b) *The Burr VI generated family of distributions is of type II, if we take*

$$W(G(x;\xi))=log\left(\frac{G(x;\xi)^{\alpha}}{1-G(x;\xi)^{\alpha}}\right)$$

*for some α* > 0

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